

--	--	--	--	--	--	--	--	--	--

# MULTIMEDIA UNIVERSITY

## FINAL EXAMINATION

TRIMESTER 2, 2019/2020

**ETN3096 – DIGITAL SIGNAL PROCESSING**  
(CE, EE, LE, OPE, TE)

5 MARCH 2020  
2:30 PM – 4:30 PM  
(2 Hours)

---

### INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 12 pages (including this cover page) with 4 Questions only and an appendix.
2. Attempt **ALL** questions. The distribution of the marks for each question is given.
3. Please print all your answers in the Answer Booklet provided.

### Question 1

- (a) What is aliasing? State the condition to ensure that a DSP system will not suffer from aliasing. [3 marks]
- (b) Consider a discrete-time system  $y[n] = ax^2[n+1]$ . Determine, with justification, if the system is
- (i) Linear [4 marks]
  - (ii) Time-invariant [4 marks]
  - (iii) Causal [2 marks]
- (c) In the system shown in Figure Q1,  $h_1[n] = [\dots 0 \ 5 \ \underline{2} \ -1 \ 1 \ 0 \ \dots]$  and  $h_2[n] = 2\delta[n] - \delta[n-2]$ , where the underlined value corresponds to  $n = 0$ .

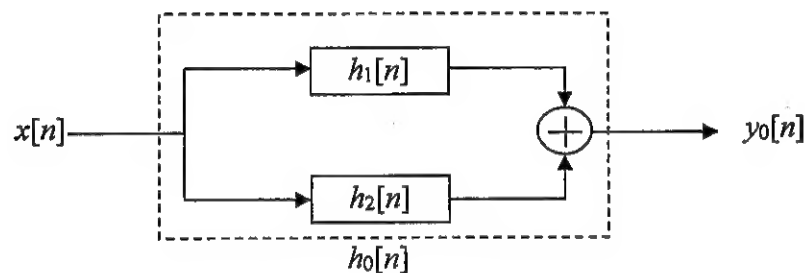


Figure Q1

- (i) Determine the impulse response of the overall system  $h_0[n]$ . [3 marks]
- (ii) For input  $x[n] = 2n(u[n+1] - u[n-2])$ , determine  $y_0[n]$  for  $n = -2$  to  $3$ . [9 marks]

Continued ...

**Question 2**

- (a) A causal linear time-invariant (LTI) system is described by a difference equation given by  $y[n] = 0.7y[n-1] - 0.12y[n-2] + x[n] + x[n-1]$ , where  $x[n]$  is the system input and  $y[n]$  is the output. All initial conditions are zero.
- (i) Determine the system function  $H(z)$ . [2 marks]
  - (ii) Comment on the system stability. [3 marks]
  - (iii) Determine the system impulse response  $h[n]$ . [5 marks]
- (b) (i) Compute the 4-point discrete Fourier transform (DFT) of  $x[n] = [4 \ 2 \ 1 \ 0]$  using direct DFT. [9 marks]
- (ii) Assume that a complex addition takes  $1\mu\text{s}$  and a complex multiplication takes  $2\mu\text{s}$ . Compare the time required for the direct evaluation of a 256-point DFT and the time required for the evaluation of a 256-point DFT through fast Fourier transform (FFT). [6 marks]

**Question 3**

- (a) Design a 5-tap finite impulse response lowpass filter with a cutoff frequency of 4000 Hz and a sampling rate of 10,000 Hz.
- (i) Calculate the filter coefficients when a rectangular window function is used. [10 marks]
  - (ii) Determine the transfer function and difference equation of the filter. [6 marks]
  - (iii) Find the gain of the filter at dc (frequency = 0). [2 marks]

**Continued ...**

**Question 3 (continued)**

(b) Adaptive filters play an important role in modern digital signal processing applications.

(i) How does adaptive filter differ from conventional digital filter? [3 marks]

(ii) Give two advantages of adaptive filter over conventional digital filter. [4 marks]

(c) Consider the digital signal processing system for noise cancellation using an adaptive filter with two coefficients shown in Figure Q3. The weight update equation is given by

$$w_i(n) = w_i(n-1) + 2\mu e(n)x(n-i), \text{ for } i = 0, 1$$

Assume that  $w_0(-1) = 0$ ,  $w_1(-1) = 0$ ,  $x(-1) = 0$ , and the convergence factor  $\mu = 0.05$ . Apply adaptive filtering to obtain outputs  $y(n)$  and  $e(n)$  for  $n = 0, 1, 2$ , as well as filter weights  $w_0(n)$  and  $w_1(n)$  for  $n = 0, 1$ , given inputs  $d(n)$  and  $x(n)$  as shown in Table Q3. Copy the table in your answer sheet. Show your calculations.

[8 marks]

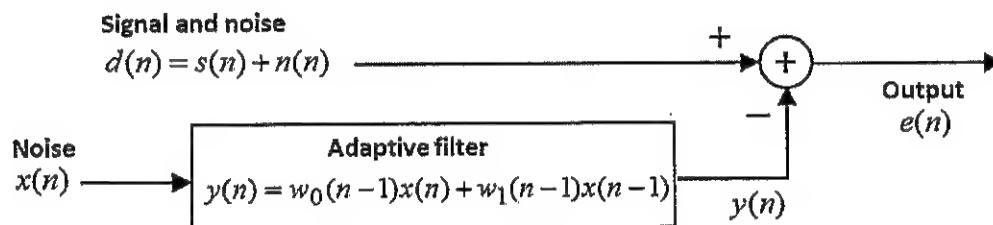


Figure Q3

Table Q3

Iteration $n$	Signal corrupted with noise $d(n)$	Noise signal $x(n)$	Filter output $y(n)$	Error signal $e(n)$	Filter weight $w_0(n)$	Filter weight $w_1(n)$
0	-1.5	-1				
1	-1	1				
2	1	-1				

Continued ...

**Question 4**

- (a) The transfer function of an infinite impulse response (IIR) filter is given by

$$H(z) = \frac{(1 - 0.1z^{-1})(1 - 0.2z^{-1})(1 - 0.4z^{-1})}{(1 - 0.3z^{-1})(1 - 0.5z^{-1})(1 - 0.6z^{-1})}.$$

- (i) Draw the Direct Form II realization of the filter. [8 marks]  
(ii) For the realization in (i), evaluate the effects of quantizing the filter coefficients to 2 decimal places. (Note that no calculations are required here.) [4 marks]

- (b) A finite impulse response (FIR) filter is described by its impulse response

$$h[n] = \frac{1}{n+1}(u[n] - u[n-3]).$$

Draw the Direct Form realization of the filter.

[5 marks]

Continued ...

## APPENDIX

## Discrete-time Fourier transform

## Properties

Property	$x[n], y[n]$	$X(e^{j\Omega}), Y(e^{j\Omega})$
Linearity	$ax[n] + by[n]$	$aX(e^{j\Omega}) + bY(e^{j\Omega})$
Time shifting	$x[n - n_0]$	$e^{-j\Omega n_0} X(e^{j\Omega})$
Frequency shifting	$e^{j\Omega_0 n} x[n]$	$X(e^{j(\Omega - \Omega_0)})$
Time reversal	$x[-n]$	$X(e^{-j\Omega})$
Differentiation	$n^k x[n]$	$(j)^k \frac{d^k}{d\Omega^k} X(e^{j\Omega})$
Convolution	$x[n] * y[n]$	$X(e^{j\Omega}) Y(e^{j\Omega})$
Multiplication	$x[n] y[n]$	$\frac{1}{2\pi} (X(e^{j\theta}) * Y(e^{j\Omega}))$

## Common pairs

$x[n]$	$X(e^{j\Omega})$
$\delta[n]$	1
$\delta[n - n_0]$	$e^{-j\Omega n_0}$
1	$\sum_{k=-\infty}^{\infty} 2\pi \delta(\Omega + 2\pi k)$
$e^{j\Omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi \delta(\Omega - \Omega_0 + 2\pi k)$
$u[n]$	$\frac{1}{1 - e^{-j\Omega}} + \sum_{k=-\infty}^{\infty} \pi \delta(\Omega + 2\pi k)$
$a^n u[n],  a  < 1$	$\frac{1}{1 - ae^{-j\Omega}}$
$(n+1)a^n u[n],  a  < 1$	$\frac{1}{(1 - ae^{-j\Omega})^2}$
$\frac{\sin \Omega_c n}{\pi n}$	$X(e^{j\Omega}) = \begin{cases} 1, &  \Omega  < \Omega_c \\ 0, & \Omega_c <  \Omega  \leq \pi \end{cases}$
$x[n] = \begin{cases} 1, & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$	$\frac{\sin \Omega (M+1)/2}{\sin \Omega/2} e^{-j\Omega M/2}$

## z-transform

### Properties

Properties	Sequence	z-transform	ROC
	$x[n]$	$X(z)$	$R_x$
	$x_1[n]$	$X_1(z)$	$R_{x1}$
	$x_2[n]$	$X_2(z)$	$R_{x2}$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	Contains $R_{x1} \cap R_{x2}$
Time shifting	$x[n-m]$	$z^{-m} X(z)$	$R_x$ except for the possible addition or deletion of the origin or infinity.
Multiplication by an exponential sequence	$a^n x[n]$	$X(z/a)$	$ a R_x$
Differentiation	$nx[n]$	$-z \frac{dX(z)}{dz}$	$R_x$ except for the possible addition or deletion of the origin or infinity.
Conjugate	$x^*[n]$	$X^*(z^*)$	$R_x$
Time reversal	$x[-n]$	$X(z^{-1})$	$1/R_x$
Convolution	$x_1[n] * x_2[n]$	$X_1(z) X_2(z)$	Contains $R_{x1} \cap R_{x2}$

### Common pairs

Sequence	Transform	ROC
$\delta[n]$	1	All $z$
$u[n]$	$\frac{1}{1 - z^{-1}}$	$ z  > 1$
$-u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$ z  < 1$
$\delta[n - m]$	$z^{-m}$	All $z$ except 0 (if $m > 0$ ) or $\infty$ (if $m < 0$ )
$a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$ z  >  a $
$-a^n u[-n - 1]$	$\frac{1}{1 - az^{-1}}$	$ z  <  a $
$na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z  >  a $
$-na^n u[-n - 1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z  <  a $

## Finite Impulse Response (FIR) Filters

Ideal impulse responses for standard FIR filters

Filter Type	Ideal Impulse Response $h(n)$
Lowpass:	$h(n) = \begin{cases} \frac{\Omega_c}{\pi}, & n = 0 \\ \frac{\sin(\Omega_c n)}{n\pi}, & n \neq 0 \end{cases}$
Highpass:	$h(n) = \begin{cases} \frac{\pi - \Omega_c}{\pi}, & n = 0 \\ \frac{-\sin(\Omega_c n)}{n\pi}, & n \neq 0 \end{cases}$
Bandpass:	$h(n) = \begin{cases} \frac{\Omega_H - \Omega_L}{\pi}, & n = 0 \\ \frac{\sin(\Omega_H n) - \sin(\Omega_L n)}{n\pi}, & n \neq 0 \end{cases}$
Bandstop:	$h(n) = \begin{cases} \frac{\pi - (\Omega_H - \Omega_L)}{\pi}, & n = 0 \\ \frac{-\sin(\Omega_H n) + \sin(\Omega_L n)}{n\pi}, & n \neq 0 \end{cases}$

FIR filter length estimation using window functions

Window Type	Window Function $w(n), -M \leq n \leq M$
Rectangular	1
Hanning	$0.5 + 0.5 \cos\left(\frac{n\pi}{M}\right)$
Hamming	$0.54 + 0.46 \cos\left(\frac{n\pi}{M}\right)$
Blackman	$0.42 + 0.5 \cos\left(\frac{n\pi}{M}\right) + 0.08 \cos\left(\frac{2n\pi}{M}\right)$

Normalized Transition Width $\Delta f = \frac{ f_{stop} - f_{pass} }{f_{sampling}}$			
Type of Window	Window Length $N$	Stopband Attenuation (dB)	Passband Ripple (dB)
Rectangular	$N = 0.9/\Delta f$	21	0.7416
Hanning	$N = 3.1/\Delta f$	44	0.0546
Hamming	$N = 3.3/\Delta f$	53	0.0194
Blackman	$N = 5.5/\Delta f$	74	0.0017



## Bilinear Transformation (BLT)

### 1. Frequency prewarping

Let  $\omega_a$  denote the analog frequency marked on the  $j\omega$ -axis on the  $s$ -plane, and  $\omega_d$  denote the digital frequency marked on the unit circle in the  $z$ -plane.

For the lowpass filter and highpass filter:

$$\omega_a = \frac{2}{T} \tan\left(\frac{\omega_d T}{2}\right)$$

For the bandpass filter and bandstop filter:

$$\omega_{al} = \frac{2}{T} \tan\left(\frac{\omega_l T}{2}\right), \quad \omega_{ah} = \frac{2}{T} \tan\left(\frac{\omega_h T}{2}\right)$$

and  $\omega_0 = \sqrt{\omega_{al}\omega_{ah}}$ ,  $W = \omega_{ah} - \omega_{al}$ .

### 2. Prototype transformation using the lowpass prototype $H_P(s)$

From lowpass to lowpass:  $H(s) = H_P(s) \Big|_{s = \frac{s}{\omega_a}}$

From lowpass to highpass:  $H(s) = H_P(s) \Big|_{s = \frac{\omega_a}{s}}$

From lowpass to bandpass:  $H(s) = H_P(s) \Big|_{s = \frac{s^2 + \omega_0^2}{sW}}$

From lowpass to bandstop:  $H(s) = H_P(s) \Big|_{s = \frac{sW}{s^2 + \omega_0^2}}$

where  $\omega_a$  denotes the analog frequency,  $\omega_0 = \sqrt{\omega_{al}\omega_{ah}}$ ,  $W = \omega_{ah} - \omega_{al}$ .

### 3. Substitute the BLT to obtain the digital filter

$$H(z) = H(s) \Big|_{s = \frac{2}{T} \frac{z-1}{z+1}}$$

## Conversion from Analog Filter Specifications to Lowpass Prototype Specifications

### Analog Filter Specifications    Lowpass Prototype Specifications

Lowpass:  $\omega_{ap}, \omega_{as}$

$$v_s = \frac{\omega_{as}}{\omega_{ap}}$$

Highpass:  $\omega_{ap}, \omega_{as}$

$$v_s = \frac{\omega_{ap}}{\omega_{as}}$$

Bandpass:  $\omega_{apl}, \omega_{aph}, \omega_{asl}, \omega_{ash}$

$$\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}, \omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$$

$$v_s = \frac{\omega_{ash} - \omega_{asl}}{\omega_{aph} - \omega_{apl}}$$

Bandstop:  $\omega_{apl}, \omega_{aph}, \omega_{asl}, \omega_{ash}$

$$\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}, \omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$$

$$v_s = \frac{\omega_{aph} - \omega_{apl}}{\omega_{ash} - \omega_{asl}}$$

$\omega_{ap}$ , passband frequency edge;  $\omega_{as}$ , stopband frequency edge

$\omega_{apl}$ , lower cutoff frequency in passband;  $\omega_{aph}$ , upper cutoff frequency in passband

$\omega_{asl}$ , lower cutoff frequency in stopband;  $\omega_{ash}$ , upper cutoff frequency in stopband

$\omega_0$ , geometric center frequency

### Closed-Form Expression for Some Useful Series

$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$	$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2}$
$\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$	$\sum_{n=0}^{N-1} n^2 = \frac{1}{6}N(N-1)(2N-1)$
$\sum_{n=0}^{N-1} n = \frac{1}{2}N(N-1)$	$\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1-1} - a^{N_2}}{1-a}$
$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$	$ a  < 1$

### Digital Butterworth and Chebyshev Filter Designs

With the given passband ripple  $A_p$  dB at the normalized passband frequency edge  $v_p = 1$ , and the stopband attenuation  $A_s$  dB at the normalized stopband frequency edge  $v_s$ ,  $\varepsilon$  is the absolute ripple specification

$$\varepsilon^2 = 10^{0.1A_p} - 1$$

Butterworth lowpass prototype order

$$n \geq \frac{\log_{10} \left( \frac{10^{0.1A_s} - 1}{\varepsilon^2} \right)}{2 \log_{10}(v_s)}$$

Chebyshev lowpass prototype order

$$n \geq \frac{\cosh^{-1} \left( \sqrt{\frac{10^{0.1A_s} - 1}{\varepsilon^2}} \right)}{\cosh^{-1}(v_s)}$$

where  $\cosh^{-1}(x) = \ln(x + \sqrt{x^2 - 1})$

3-dB Butterworth Lowpass Prototype Transfer Functions ( $\varepsilon = 1$ )

$n$	$H_p(s)$
1	$\frac{1}{s+1}$
2	$\frac{1}{s^2 + 1.4142s + 1}$
3	$\frac{1}{s^3 + 2s^2 + 2s + 1}$

Chebyshev Lowpass Prototype Transfer Functions with 1dB Ripple ( $\varepsilon = 0.5088$ )

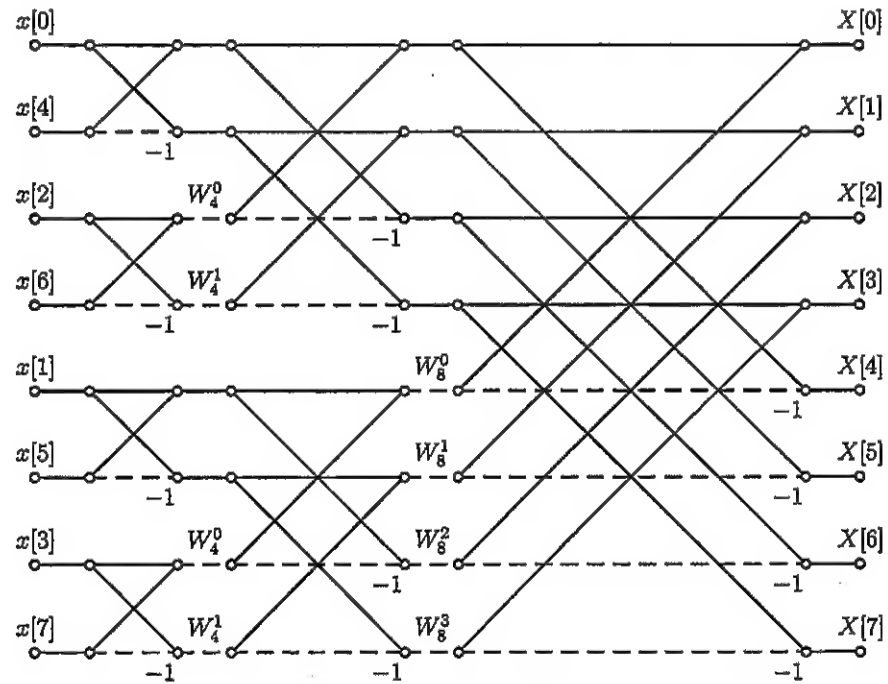
$n$	$H_p(s)$
1	$\frac{1.9652}{s + 1.9652}$
2	$\frac{0.9826}{s^2 + 1.0977s + 1.1025}$
3	$\frac{0.4913}{s^3 + 0.9883s^2 + 1.2384s + 0.4913}$

## Discrete Fourier Transform

### Properties

Property	$x[n]$	$X[k]$
Linearity	$A_1x_1[n] + A_2x_2[n]$	$A_1X_1[k] + A_2X_2[k]$
Time shifting	$x[\langle n - n_0 \rangle_N]$	$X[k]W_N^{kn_0}$
Frequency shifting	$x[n]W_N^{-k_0n}$	$X[\langle k - k_0 \rangle_N]$
Time reversal	$x[\langle -n \rangle_N]$	$X[\langle -k \rangle_N]$
Conjugation	$x^*[n]$	$X^*[\langle -k \rangle_N]$
Convolution	$x[n] \otimes y[n]$	$X[k]Y[k]$
Modulation	$Nx[n]y[n]$	$X[k] \otimes Y[k]$

### The decimation-in-time fast Fourier transform



End of Paper



